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## **The Calculation of Hydrodynamic Coefficients for Underwater Vehicles**

D.A. Jones, D.B. Clarke,  
I.B. Brayshaw, J.L. Barillon and  
B. Anderson

DSTO-TR-1329

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*D.A. Jones, D.B. Clarke, I.B. Brayshaw, J.L. Barillon, and B. Anderson*

**Maritime Platforms Division  
Platforms Sciences Laboratory**

DSTO-TR-1329

## ABSTRACT

Maritime Platforms Division within DSTO is currently studying the emerging science and technology of autonomous underwater vehicles for defence applications. As part of an examination of the requirements for the hydrodynamics and manoeuvrability of these vehicles MPD has been tasked with the development of models to determine the hydrodynamic coefficients of simple and complex submerged bodies as a function of their shape. These coefficients are specific to the vehicle and provide the description of the hydrodynamic forces and moments acting on the vehicle in its underwater environment. This report provides a detailed discussion and evaluation of three of the existing methods which have been documented in the literature for the calculation of these coefficients. Sample calculations using some of these techniques are presented, and the accuracy and applicability of these calculational methods to the underwater vehicles of interest to the DSTO are described. It is concluded that none of the methods surveyed has the necessary generality to encompass all the shapes of interest to DSTO work, and alternative computational techniques are recommended which should allow the hydrodynamic coefficients of more complex underwater vehicles to be determined.

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# The Calculation of Hydrodynamic Coefficients for Underwater Vehicles

## Executive Summary

Autonomous unmanned underwater vehicles (UUVs) are emerging as a significant capability enhancer for future generation submarines. The hydrodynamic performance of UUVs is an area of interest having implications for control, navigation, launch and recovery, energy requirements and payload. A useful tool for gaining an understanding of the performance of a UUV is a dynamic simulation of the equations of motion of the vehicle. To perform these simulations the hydrodynamic coefficients of the vehicle must first be provided. These coefficients are specific to the vehicle and provide the description of the hydrodynamic forces and moments acting on the vehicle in its underwater environment. This report provides a detailed discussion and evaluation of three of the existing methods which have been documented in the literature for the calculation of these coefficients.

Before a detailed description of each method is provided a clear definition of these hydrodynamic coefficients is given and the significance of each of the various coefficients is discussed. It is concluded that in the longitudinal plane there are only five linear hydrodynamic derivatives of any significance. Simplified derivations for each of these five coefficients are then provided which highlight the physical significance of each term.

A detailed description of the calculation of each of these five coefficients using three different methods documented in the literature is then given. Two of these methods (the U.S. Air Force DATCOM method and the Roskam method) are based on techniques developed in the aeronautical industry, while the third is based on methods applicable to the calculation of the coefficients of single screw submarines and was developed at University College, London University. One of these methods (the DATCOM method) was then used to calculate the hydrodynamic coefficients for four different torpedo shapes and the calculated values were then compared with experimental results. It was concluded that the methods described above could only calculate accurate values of the hydrodynamic coefficients for specific vehicle shapes, and that a more promising method would be to combine experimental measurements on scaled models with current Computational Fluid Dynamics capabilities.

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# 1. Introduction

Autonomous unmanned underwater vehicles (UUVs) are emerging as a significant capability enhancer in concepts of future generation submarines. While this is widely recognised, the assessment of the performance of these vehicles in operational scenarios needs investigation before these concepts will be considered. The hydrodynamic performance of UUVs is an area of interest having implications for control, navigation, launch and recovery, energy requirements and payload. In fact an understanding of the hydrodynamic performance of a UUV is essential to its capability to perform a mission. A useful tool for performing these evaluations is a dynamic simulation where the hydrodynamic characteristics of the vehicle are characterised in the equations of motion.

Simulation of the motion of an underwater vehicle requires the numerical solution of six coupled non-linear differential equations. Three of these equations describe the translational motion of the vehicle, while the remaining three equations describe rotational motion of the vehicle about some fixed point on the body. This fixed point is usually taken to be either the centre of mass (CM), or the centre of buoyancy (CB) of the vehicle. Detailed derivations and discussions of these equations of motion can be found in many references. The report by Rocard [1] is perhaps the most relevant to the work described here, but detailed descriptions can also be found in the work of Abkowitz [2], Anderson [3], Brutzman [4], and Gertler and Hagen [5]. Strumpf [6] provides an extension of these equations by considering the equations of motion of a submerged body with varying mass.

Software packages for the solution of these equations are either readily available, or are relatively easy to develop. Packages currently in use by the DSTO include UUV6DOF, a dynamic six degree of freedom Matlab/Simulink code for Unmanned Underwater Vehicles developed in conjunction with the Australian Maritime Engineering CRC, and UUVSIM, a six degree of freedom model for navigation, guidance and control developed by DERA<sup>1</sup> and exchanged under The Technical Cooperation Programme agreement. Before these packages can be used to simulate the motion of an underwater vehicle certain hydrodynamic coefficients must be provided. These coefficients are specific to the vehicle and provide the description of the hydrodynamic forces and moments acting on the vehicle in its underwater environment. The evaluation of these hydrodynamic coefficients is a non-trivial exercise, and the purpose of this report is to describe some of the existing methods for the calculation of these coefficients which have been documented in the literature, and to discuss the applicability of these calculation methods to the underwater vehicles of interest to the DSTO.

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<sup>1</sup> DERA has since been replaced by the government laboratory DSTL and the private company QinetiQ.

## 2. Definition of Hydrodynamic Coefficients

The terms in the equations of motion which represent the hydrodynamic forces and moments acting on the vehicle are often expanded in a Taylor series about some convenient reference condition. For aircraft and surface ships this reference condition is usually taken to be the equilibrium condition of forward motion at constant speed  $U_o$ . This approach has also been adopted for the analysis of UUVs although, for reasons which will be discussed later, it is less appropriate for these types of vehicles. A rectangular cartesian coordinate system attached to the vehicle has been used in this report. The origin of the coordinate system is located at the centre of gravity of the vehicle, the x axis lies along the longitudinal axis of the vehicle with the positive direction facing forward. The y axis points towards starboard, and the direction of the z axis is determined by the right hand rule and points downwards. The three components of the hydrodynamic force along the directions x, y and z are denoted by X, Y and Z respectively, and the three components of the hydrodynamic torque by L, M, and N. This is illustrated in Figure 1. The path of the vehicle is then assumed to be intentionally altered slightly by deflection of various control surfaces on the vehicle. The forward translational velocity of the vehicle now has a value  $U = U_o + u$ , and the vehicle also acquires components of translational velocity in the y and z directions, denoted by v and w respectively. The fundamental approximation of the approach adopted here is that

$$|u|, |v|, |w| \ll |U_o| \quad (2.1)$$

The vehicle may also acquire angular velocities p, q, and r about the x, y, and z axes respectively, and a similar assumption is made about the magnitude of these angular velocities, ie.

$$|p l_{REF} / U_o|, |q l_{REF} / U_o|, |r l_{REF} / U_o| \ll 1 \quad (2.2)$$

where  $l_{REF}$  is some reference length on the vehicle. The three components of force, X, Y, and Z, and the three components of the torque, L, M, and N are then expanded up to first order terms in the linear velocities u, v, and w, and the angular velocities p, q and r, where these velocities now represent perturbations to the equilibrium condition of steady state forward motion. Note that any dependence on linear or angular accelerations, and any higher order terms in the velocities, are neglected. The expressions for the forces and moments then take the form:

$$X = X_o + X_u u + X_v v + X_w w + X_p p + X_q q + X_r r \quad (2.3)$$

$$Y = Y_o + Y_u u + Y_v v + Y_w w + Y_p p + Y_q q + Y_r r \quad (2.4)$$

$$Z = Z_o + Z_u u + Z_v v + Z_w w + Z_p p + Z_q q + Z_r r \quad (2.5)$$

$$L = L_o + L_u u + L_v v + L_w w + L_p p + L_q q + L_r r \quad (2.6)$$

$$M = M_o + M_u u + M_v v + M_w w + M_p p + M_q q + M_r r \quad (2.7)$$

$$N = N_o + N_u u + N_v v + N_w w + N_p p + N_q q + N_r r \quad (2.8)$$

In Equations (2.3) through (2.8) the subscript notation represents partial differentiation, so that  $X_u = \frac{\partial X}{\partial u}$ , and the zero subscript refers to conditions in the assumed reference state. The partial derivatives are known as hydrodynamic coefficients, hydrodynamic derivatives, or stability derivatives, and are evaluated at the reference condition.

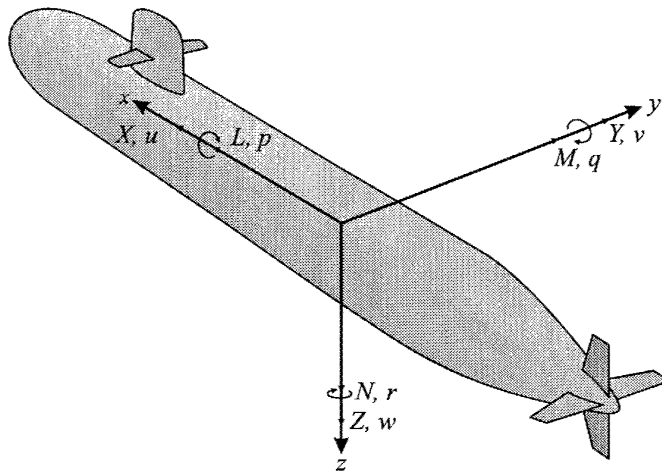


Figure 1. Schematic of the coordinate system used.

To first order therefore there are a possible 36 hydrodynamic coefficients which could be evaluated to describe the dynamics of the vehicle. If the vehicle has certain symmetries however then many of these coefficients are zero. For example if the x-z plane is a plane of symmetry, so that the vehicle has Left/Right symmetry, then terms such as  $Y_u$ ,  $Y_w$ ,  $L_u$ ,  $L_w$  etc. will all be zero.  $Y_u$  for example is the contribution to the component of force in the y direction due to motion in the x direction. For a body with Left/Right symmetry it is easy to see that this contribution will always be zero. Clarke [7] has undertaken an extensive analysis of all 36 linear hydrodynamic coefficients using detailed symmetry arguments and has applied them to typical underwater vehicle shapes. He concluded that the only non-zero coefficients for axi-symmetric UUVs are  $X_u$ ,  $X_v$ ,  $X_w$ ,  $Z_w$ ,  $Z_q$ ,  $M_w$  and  $M_q$  in the longitudinal plane, and  $Y_v$ ,  $Y_r$ ,  $N_v$  and  $N_r$  in the lateral plane.

Etkin [8], in discussing aerodynamic derivatives for aeroplanes, notes that for symmetric aircraft the derivatives of the asymmetric or lateral forces and moments,  $Y$ ,  $L$ , and  $N$ , with respect to the symmetric or longitudinal motion variables  $u$ ,  $w$  and  $q$ , will be zero. This implies that  $Y_u$ ,  $Y_w$ ,  $Y_q$ ,  $L_u$ ,  $L_w$ ,  $L_q$ ,  $N_u$ ,  $N_w$  and  $N_q$  are zero for aircraft, and bodies which exhibit similar symmetry properties. Etkin also makes the further approximation that all the derivatives of the symmetric forces ( $X$ ,  $Z$ ) and moments ( $M$ ) with respect to the asymmetric variables ( $v$ ,  $p$  and  $r$ ) can be neglected. This implies that  $X_v$ ,  $X_p$ ,  $X_r$ ,  $Z_v$ ,  $Z_p$ ,  $Z_r$ ,  $M_v$ ,  $M_p$  and  $M_r$  are zero. The above considerations eliminate 18 of the 36 derivatives, and the expressions for the forces and moments now become

$$X = X_o + X_u u + X_w w + X_q q \quad (2.9)$$

$$Y = Y_o + Y_v v + Y_p p + Y_r r \quad (2.10)$$

$$Z = Z_o + Z_u u + Z_w w + Z_q q \quad (2.11)$$

$$L = L_o + L_v v + L_p p + L_r r \quad (2.12)$$

$$M = M_o + M_u u + M_w w + M_q q \quad (2.13)$$

$$N = N_o + N_v v + N_p p + N_r r \quad (2.14)$$

Similar symmetry arguments are also described by Abkowitz [2] while discussing the stability of ocean vehicles. In Appendix I of reference [2] for example he showed that the terms  $X_v$ ,  $X_p$  and  $X_r$  are all zero if port/starboard symmetry is assumed. Russell [9] also considered the effect of symmetry on the stability derivatives for aircraft and showed that the assumption that the aircraft has symmetry about a vertical plane implies that half the stability derivatives can be taken to be zero.

There are still 18 first order derivatives to be considered, 9 in the longitudinal (or vertical) plane,  $X_u$ ,  $X_w$ ,  $X_q$ ,  $Z_u$ ,  $Z_w$ ,  $Z_q$ ,  $M_u$ ,  $M_w$  and  $M_q$ , and 9 in the lateral (or horizontal) plane,  $Y_v$ ,  $Y_p$ ,  $Y_r$ ,  $L_v$ ,  $L_p$ ,  $L_r$ ,  $N_v$ ,  $N_p$ , and  $N_r$ . Several of these, particularly in the longitudinal plane, can also be neglected. Blakelock [10], for example, noted that  $X_q$  describes the effect of pitch rate on drag, and can be neglected. Russell [9] also stated that  $X_q$  is usually neglected. Brayshaw [11] adapted the methods of Roskam [12] for the calculation of aerodynamic derivatives of aircraft to the calculation of hydrodynamic derivatives for underwater vehicles, and also came to the conclusion that  $X_q$  can be neglected. Brayshaw's analysis also concluded that  $X_w$ ,  $M_u$  and  $Z_u$  can be taken to be zero for an underwater vehicle when the vehicle generates no lift in its steady state or reference condition. For both aeroplanes and underwater vehicles the reference state is usually taken to be forward motion at constant velocity. In this case, for an AUV, the assumption of zero lift in the steady state is probably justified. For a towed underwater vehicle however this assumption is probably incorrect. A towed UUV will probably generate negative lift to

counteract the vertically upward component of force provided by the towing cable. In this case the assumption that  $X_w$ ,  $M_u$  and  $Z_u$  can be set to zero may need to be modified.

Combining the analyses of Etkin [8] and Russell [9] for aircraft with that of Brayshaw [11] for underwater vehicles, the conclusion can be made that in the longitudinal plane there are only five linear hydrodynamic derivatives of any significance:  $X_u$ ,  $Z_w$ ,  $Z_q$ ,  $M_w$  and  $M_q$ . This conclusion is also in agreement with Peterson's work [13]. In his HYCOF subroutine for the calculation of the linear and nonlinear hydrodynamic coefficients of a submersible the only linear coefficients calculated are those listed above.

Strumpf [6] also considered each of the linear hydrodynamic coefficients in detail and provides a summary of the relative significance of each of the terms. He noted that  $X_v$ ,  $X_p$ ,  $X_r$ ,  $Z_v$ ,  $Z_p$ ,  $Z_r$ ,  $M_v$ ,  $M_p$ ,  $M_r$ ,  $Y_u$ ,  $Y_w$ ,  $Y_q$ ,  $L_u$ ,  $L_w$ ,  $L_q$ ,  $N_u$ ,  $N_w$ , and  $N_q$  can be set equal to zero on the basis of symmetry arguments. On the basis of experimental results he stated that  $X_u$  is an important coefficient, while  $X_w$  and  $X_q$  can be neglected. Similarly, based on experimental results,  $Z_w$  and  $Z_q$  are considered to be important coefficients, while  $Z_u$  is considered to be less important. For the longitudinal component of torque  $M$  experimental results showed that both  $M_w$  and  $M_q$  are important, while  $M_u$  is less significant. Hence, for the longitudinal coefficients Strumpf considered that  $X_u$ ,  $Z_w$ ,  $Z_q$ ,  $M_w$ , and  $M_q$  were important coefficients, while  $Z_u$  and  $M_u$  were less significant. The remaining longitudinal coefficients were taken to be zero, either because of symmetry considerations, or on the basis of experimental results. These results are in good agreement with the considerations discussed above, where it was concluded that the only significant longitudinal coefficients were  $X_u$ ,  $Z_w$ ,  $Z_q$ ,  $M_w$  and  $M_q$ , while  $M_u$ ,  $Z_u$  and  $X_w$  would only be non-zero if the vehicle had net lift in the steady state reference condition. For the lateral coefficients Strumpf concluded from experimental results that  $Y_v$ ,  $Y_r$ ,  $N_v$ , and  $N_r$  are significant terms. There is little experimental data available on  $Y_p$ ,  $L_p$  and  $N_p$ , although  $L_p$  is important if banked turns are considered to be important. Strumpf considered  $L_v$  and  $L_r$  to be of less significance, and the remaining terms to be zero due to symmetry considerations.

There are relatively few reports available in the literature which describe methods for the calculation of linear longitudinal and lateral hydrodynamic coefficients based on geometric parameters. The report by Peterson [14] is one of the most comprehensive. This provides a description and comparison of seven widely used semi-empirical methods for predicting several important linear hydrodynamic coefficients for conventional marine vehicles. The coefficients considered are the four longitudinal hydrodynamic derivatives  $Z_w$ ,  $M_w$ ,  $Z_q$  and  $M_q$  for the bare hull, and the two coefficients  $Z_w$  and  $M_w$  for the bare hull plus tail configuration. The seven methods are compared by applying them to three torpedoes and three submersibles for which experimental data are available.

Another useful reference is a set of University College London Postgraduate submarine design notes [15]. This provides a very detailed example of how to calculate hydrodynamic derivatives for a single screw submarine. The method described assumes

that the derivatives for the complete submarine can be found by adding the contributions of each of the components (hull, propeller, appendages) and including any interference effects between components. The longitudinal derivatives which are calculated are again  $Z_w$ ,  $M_w$ ,  $Z_q$  and  $M_q$ .

Nahon [16] describes how to determine underwater vehicle hydrodynamic derivatives using the USAF Datcom method. This method is the same as the first method described by Peterson [14]. Nahon illustrated the method by using it to calculate the three hydrodynamic derivatives  $M_w$ ,  $Y_v$  and  $N_v$  for the ARCS (Autonomous Remotely Controlled Submersible) underwater vehicle. In a more recent paper [17] Nahon describes a simplified method for calculating the dynamics of autonomous underwater vehicles. In this method the hydrodynamic derivatives are avoided by calculating the hydrodynamic forces directly from known relations which govern the flow around simple shapes. The method is explained and illustrated by application to the ARCS vehicle. Basically, lift and drag forces are defined for the main hull, and any additional control surfaces (fins, rudder, etc.). The lift and drag forces are then resolved as force components in the body frame by transforming them through the pitch and yaw angles, as appropriate. The total force and moment acting on the vehicle are then determined through a summation of the component effects, with correction factors to account for interference effects. The performance of the model was then analysed through a simulation study of the ARCS vehicle's motion in a representative manoeuvre, and the simulation results were very close to those measured. Whilst this appears to be an intuitively appealing approach to the dynamics of AUVs, as it only requires specification of the vehicle's geometry, and the lift and drag characteristics of its constituent elements, the methods used to take into account the interference effects are relatively rudimentary. Despite this, the simulated motion agrees well with the experimental results.

The reports by Wolkerstorfer [18] and Holmes [19] from the Naval Postgraduate School in Monterey illustrate the application of the DATCOM method described by Peterson [14] to the calculation of hydrodynamic derivatives for a linear manoeuvring model for the simulation of SLICE hulls, as well as the prediction of hydrodynamics coefficients utilising geometric considerations. In the latter report the hull shape considered is a body of revolution having a basic submarine shape. The nose is elliptical, the mid body is cylindrical, and the base is conical. The aim of the work was to modify the body shape slightly to see how the geometric changes affected the hydrodynamic derivatives. It should be noted that only the coefficients  $Y_v$ ,  $N_v$ ,  $Y_r$ , and  $N_r$  are calculated in this application of the DATCOM method. A further point to note is that the DATCOM method assumes that the vehicle has rotational symmetry about the longitudinal axis, and so the formulae used to calculate  $Y_v$ ,  $N_v$ ,  $Y_r$ , and  $N_r$  are the same as those used to calculate  $Z_w$ ,  $M_w$ ,  $Z_q$  and  $M_q$ . The only other reference to the calculation of lateral hydrodynamic coefficients which has been found is Lewis [20]. Section 9 of chapter 9 is titled "Theoretical Prediction of Hydrodynamic Coefficients and Systems Identification", and contains a good discussion of methods used to calculate  $Y_v$ ,  $Y_r$ ,  $N_v$ , and  $N_r$ .

### 3. Derivation of Simplified Expressions for Hydrodynamic Derivatives

Before considering some of the more detailed methods presented in the literature for evaluating the hydrodynamic coefficients of particular underwater vehicles, the relatively simpler task of calculating coefficients for an isolated lifting surface attached as an appendage to a larger body is considered. This enables an understanding of the physical significance of each of the important hydrodynamic coefficients to be obtained. The longitudinal and lateral coefficients are considered separately, although the methods for each set of coefficients are essentially the same.

#### 3.1 Longitudinal Coefficients

Figure 2 shows the lift and drag forces in the longitudinal plane (the x-z plane) acting on an isolated wing, and the resolution of these forces into components along the x and z axes. From Figure 2

$$X = L \sin \alpha - D \cos \alpha \quad (3.1)$$

$$Z = -L \cos \alpha - D \sin \alpha \quad (3.2)$$

where L represents the lift force, which is perpendicular to the direction of the wind flow, and D represents the drag force, which is parallel to the wind flow.

Equations (3.1) and (3.2) are usually written in terms of the lift coefficient  $C_L$  and drag coefficient  $C_D$ , which are defined by the expressions  $C_L = L / (\frac{1}{2}\rho V^2 S_{ref})$ , and  $C_D = D / (\frac{1}{2}\rho V^2 S_{ref})$ , where  $\rho$  is the density of the fluid medium,  $S_{ref}$  is a reference area, usually the planform area of the wing for an aeroplane, and V is the magnitude of the flow velocity, which is given by the expression  $V^2 = (U_0 + u)^2 + v^2 + w^2$ .

Equations (3.1) and (3.2) now take the form

$$X = \frac{1}{2}\rho V^2 S_{ref} (C_L \sin \alpha - C_D \cos \alpha) \quad (3.3)$$

$$Z = -\frac{1}{2}\rho V^2 S_{ref} (C_L \cos \alpha + C_D \sin \alpha) \quad (3.4)$$

Before proceeding to derive expressions for the longitudinal coefficients  $X_u$ ,  $Z_w$ ,  $Z_q$ ,  $M_w$  and  $M_q$  it should be pointed out that values for the hydrodynamic coefficients are usually quoted in dimensionless form. This can cause confusion because there is no universally accepted convention for making the coefficients non-dimensional. In particular, there are significant differences between the conventions adopted by the aeronautical and underwater communities, and even differences between the British and American aeronautical conventions. In this report a *non-dimensional* derivative is denoted by the prime notation. Hence  $Z_w$  denotes a dimensional coefficient, while  $Z'_w$  denotes the corresponding *dimensionless* coefficient. There are several conventions currently in use to

render dimensional coefficients dimensionless. Coefficients such as  $Z_w$  for example can be made non-dimensional by dividing by either  $\frac{1}{2}\rho V S_{ref}$ , which is the convention adopted by Russell [9], or by  $\rho V S_{ref}$ , which is the convention adopted by Babister [21]. There can also be confusion with the rotary coefficients, such as  $M_q$ , where the practice in the aeronautical literature is to use a factor of two when making an angular velocity non-dimensional. This convention does not seem to have been adopted by the underwater community. To avoid confusion of this type, many of the expressions in this section have been left in dimensional form.

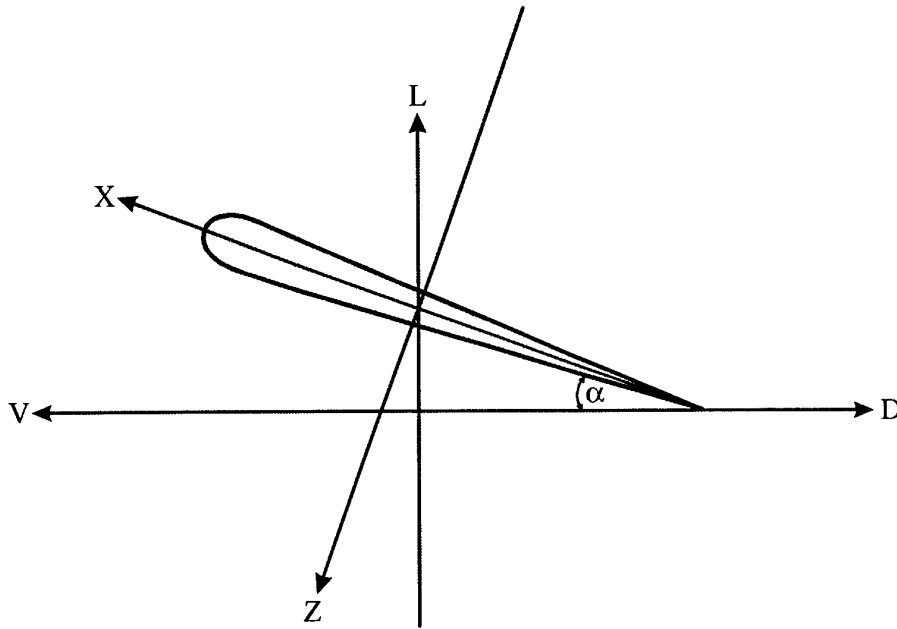


Figure 2. Lift and Drag forces in the longitudinal plane, and their resolution into components  $X$  and  $Z$  along the  $x$  and  $z$  axes.

One convention which is universally adopted however, at least when performing a linear analysis, such as that described here, is the small angle approximation. As the expansion scheme shown in Equations (2.3) to (2.8) represents a perturbation about a steady state reference state it is assumed that all angles are small, and the following approximations are made:

$$\sin \alpha \approx \alpha \quad (3.5)$$

$$\cos \alpha \approx 1 \quad (3.6)$$

With these approximations Equations (3.3) and (3.4) become:

$$X = \frac{1}{2}\rho V^2 S_{\text{ref}} (C_L \alpha - C_D) \quad (3.7)$$

$$Z = -\frac{1}{2}\rho V^2 S_{\text{ref}} (C_L + C_D \alpha) \quad (3.8)$$

From Figure 2 the following relationships can be described:

$$V^2 = (U_o + u)^2 + v^2 + w^2 \quad (3.9)$$

$$\frac{\partial V}{\partial U} = \frac{U}{V} = \cos \alpha \approx 1 \quad (3.10)$$

$$\frac{\partial V}{\partial w} = \frac{w}{V} = \sin \alpha \approx \alpha \approx 0 \quad (3.11)$$

$$\frac{\partial \alpha}{\partial U} = -\frac{\sin \alpha}{V} \approx 0 \quad (3.12)$$

$$\frac{\partial \alpha}{\partial w} = \frac{1}{U} \approx \frac{1}{V} \quad (3.13)$$

Assuming that  $X$  is a function of both  $V$  and  $\alpha$ , then by using the chain rule it can be found that

$$X_u = \frac{\partial X}{\partial u} = \frac{\partial X}{\partial U} = \frac{\partial X}{\partial V} \frac{\partial V}{\partial U} + \frac{\partial X}{\partial \alpha} \frac{\partial \alpha}{\partial U} \approx \frac{\partial X}{\partial V} \quad (3.14)$$

Now, using the standard approximations

$$\alpha = 0, \quad \frac{\partial \alpha}{\partial V} = 0, \quad \frac{\partial C_L}{\partial V} = 0, \quad \text{and} \quad \frac{\partial C_D}{\partial V} = 0 \quad (3.15)$$

the following expression for  $X_u$  can be found:

$$X_u = \frac{\partial X}{\partial V} = -\rho V S C_D \quad (3.16)$$

This expression agrees with the result quoted by Strumpf [6], Babister [21], Smetana [22], and by McCormack [23].

For the coefficient  $Z_w$  consider Equation (3.8). Differentiating with respect to  $w$  and using the above approximations leads to

$$Z_w = \frac{\partial Z}{\partial w} = \frac{\partial Z}{\partial V} \frac{\partial V}{\partial w} + \frac{\partial Z}{\partial \alpha} \frac{\partial \alpha}{\partial w} \approx \frac{1}{V} \frac{\partial Z}{\partial \alpha} \quad (3.17)$$

$$Z_w = -\frac{1}{2}\rho VS \left( \frac{\partial C_L}{\partial \alpha} + C_D \right) \quad (3.18)$$

Equation (3.18) is again a standard result and agrees with the expressions quoted by Babister [21] and Smetana [22].

The calculation of  $Z_q$  in the aeronautical literature usually proceeds along the following lines. Pitching an aeroplane at a rate  $q$  gives a downward velocity to the tail of  $w_T = q \times l_T$ , where  $l_T$  is the length of the tailplane arm, which is approximately equal to the distance between the aerodynamic centre of the tailplane and the centre of gravity of the aeroplane. This additional downward velocity can be treated in the same manner as the downward velocity considered in the derivation of  $Z_w$ . Hence Russell [9] derives the following (nondimensional) result

$$Z'_q = \frac{S_T l_T}{Sc} \left( \frac{\partial C_L^T}{\partial \alpha} + C_D^T \right) \approx -\bar{V}_T a_1 \quad (3.19)$$

Here  $S_T$  is the tailplane reference area,  $\bar{V}_T$  is the horizontal tail volume ratio, or tail volume coefficient, defined as  $S_T l_T / Sc$ , where  $S$  is the reference area for the main wing and  $c$  is the wing chord length, and  $a_1$  is the tailplane lift/curve slope. The approximation made in Equation (3.19) assumes that the tailplane lift/curve slope is much larger than the tailplane drag coefficient. This is a common assumption. Equation (3.19) is also quoted by Etkin [8] and Babister [21]. Both Etkin and Russell note that on a conventional aeroplane the tailplane provides the most significant contribution to  $Z_q$ . This is because any additional component of downward velocity imparted to the main wings via the rotational pitching velocity will be negligibly small due to the relatively close location of these wings to the centre of gravity of the aeroplane. Houghton and Carpenter [24] also provide some additional understanding of  $Z_q$ , and also  $M_q$ , by using thin wing theory to calculate  $Z_q$  and  $M_q$  for a thin aerofoil.

To derive Equation (3.19) it was assumed that the additional downward velocity at the tailplane of magnitude  $w_T$  leads to an additional component of force in the  $z$  direction,  $\delta Z_T$ , which is given by

$$\delta Z_T = Z_w^T \cdot w_T = -\frac{1}{2}\rho V S_T \left( \frac{\partial C_L^T}{\partial \alpha} + C_D^T \right) q l_T \quad (3.20)$$

Differentiating with respect to  $q$ , and then normalising with respect to  $\frac{1}{2}\rho V S_{ref} l_{ref}$ , it is found that

$$Z'_q = \frac{\partial \delta Z_T}{\partial q} / \frac{1}{2}\rho V S_{ref} l_{ref} = -\frac{S_T l_T}{S_{ref} l_{ref}} \left( \frac{\partial C_L^T}{\partial \alpha} + C_D^T \right) = -\bar{V}_T a_1 \quad (3.21)$$

which is the same as the expression given in Equation (3.19).

Another way to derive an expression for  $Z_q$  is as follows. If the aerodynamic centre is located a distance  $l_m$  along the x axis from the centre of gravity, then a pitching moment about the centre of gravity will induce a local velocity component  $w$  at the aerodynamic centre, given by  $w = q l_m$ , hence  $\partial w / \partial q = l_m$ . (Note that  $l_m$  is positive if the aerodynamic centre is aft of the centre of gravity). Equation (3.8) can then be differentiated with respect to  $q$  to obtain

$$Z_q = \frac{\partial Z}{\partial w} \frac{\partial w}{\partial q} = l_m \frac{\partial Z}{\partial w} = l_m Z_w \quad (3.22)$$

Note that the  $Z_q$  given by Equation (3.22) is in dimensional form. To put Equation (3.22) into dimensionless form, the non-dimensional form of  $Z_w$ , ie  $Z'_w$  would be used, and then the moment arm  $l_m$  would be divided by a reference length  $l_{ref}$ . Because  $Z_q$  would then represent a contribution from the tailplane, rather than the main wing, it would then be necessary to scale the coefficient by the ratio of the representative reference areas,  $S_T/S_{ref}$ . With these adjustments, Equation (3.22) then becomes identical with Equation (3.21).

If a unique aerodynamic centre for the entire vehicle was defined, Equation (3.22) would be the correct expression for  $Z_q$  for the vehicle. In practice, an aerodynamic centre is defined for each lifting surface on the vehicle and a  $Z_q$  is defined for each lifting surface.  $Z_q$  for the vehicle is then found by summing  $Z_q$  for each of the lifting surfaces. For an aeroplane the aerodynamic centre for the main wing is often located very close to the centre of gravity, so the contribution to  $Z_q$  from the main wing is often negligible compared with the contribution from the tail, which is often quite large because of the long moment arm.

The expression for the pitching moment  $M$ , assuming that the centre of gravity and the aerodynamic centre are separated by a distance  $l_m$  along the x axis and  $t_m$  along the z axis, is simply given by:

$$M = Z l_m + X t_m \quad (3.23)$$

Hence the expression for  $M_w$  is given by:

$$M_w = Z_w l_m + X_w t_m \quad (3.24)$$

$Z_w$  has already been calculated in Equation (3.18), and  $X_w$  is easily calculated from Equation (3.7):

$$X_w = \frac{\partial X}{\partial w} = \frac{\partial X}{\partial V} \frac{\partial V}{\partial w} + \frac{\partial X}{\partial \alpha} \frac{\partial \alpha}{\partial w} = \frac{1}{V} \frac{\partial X}{\partial \alpha} \quad (3.25)$$

ie. 
$$X_w = \frac{1}{2} \rho V S \left( C_L - \frac{\partial C_D}{\partial \alpha} \right) \quad (3.26)$$

Equation (3.24) is in a different form to that given by most aeronautical references. Both Babister [21] and Smetana [22], for example, quote the dimensional expression:

$$M_w = \frac{\partial M}{\partial w} = \frac{1}{V} \frac{\partial M}{\partial \alpha} = \frac{1}{2} \rho V S \eta_{\text{ref}} \frac{\partial C_m}{\partial \alpha} \quad (3.27)$$

where  $\eta_{\text{ref}}$  is a reference length, usually taken to be the mean chord length in the aeronautical literature, and  $C_m$  is the dimensionless pitching moment coefficient. This can be written as:

$$M_w = \frac{1}{2} \rho V S \eta_{\text{ref}} \frac{\partial C_m}{\partial C_L} \frac{\partial C_L}{\partial \alpha} = \frac{1}{2} \rho V S \eta_{\text{ref}} \frac{\partial C_L}{\partial \alpha} (h - h_n) \quad (3.28)$$

where  $(h - h_n)$  is the distance between the centre of gravity and the aerodynamic centre, a result which is derived in many of the aeronautical references, Babister [21], McCormack [23] and Clancy [25]. If both the centre of gravity and the aerodynamic centre lie on the x axis, so  $t_m$  is zero, and it is recognized that  $M_w$  as given by Equation (3.24) is in body-fixed axes, while the  $M_w$  as given by Equation (3.28) is in the wind-axes system, then the two expressions for  $M_w$  are identical.

A useful expression for  $M_q$  can be derived from Equation (3.23):

$$M_q = Z_q l_m + X_q t_m \quad (3.29)$$

Remembering that an approximation where  $X_q$  equals zero is being used

$$M_q = l_m^2 Z_w \quad (3.30)$$

Equation (3.30) is a simple expression for  $M_q$  which is easily evaluated, and is identical to the expression used in reference [15] to calculate  $M_q$  for submarines. Expressions for  $M_q$  in the aeronautical literature, however, are often quoted in quite a different form. For example Russell [9] uses the following expression for  $M_q$

$$M'_q = -\bar{V}_T \frac{l_T}{l_{\text{ref}}} \frac{\partial C_L^T}{\partial \alpha} \quad (3.31)$$

where  $\bar{V}_T$  is the horizontal tail volume ratio defined by  $\bar{V}_T = l_T S_T / l_{\text{ref}} S_{\text{ref}}$ .

The derivation of Equation (3.31) is similar to the derivation of the equation for  $Z_q$ . An aircraft pitching at a rate  $q$  imparts a downward velocity to the tailplane of magnitude  $w_T = ql_T$ , where  $l_T$  is the tailplane arm. It is assumed then that this downward velocity at the tailplane leads to an additional component of force in the z direction,  $\delta Z_T$ , which is given by

$$\delta Z_T = Z_w^T \cdot w_T = -\frac{1}{2} \rho V S_T \left( \frac{\partial C_L^T}{\partial \alpha} + C_D^T \right) q l_T \quad (3.32)$$

This in turn contributes to a change in the pitching moment  $\delta M$  given by

$$\delta M = \delta Z_w \cdot l_T = -\frac{1}{2} \rho V S_T \left( \frac{\partial C_L^T}{\partial \alpha} + C_D^T \right) q l_T^2 \quad (3.33)$$

which leads to

$$M_q = \frac{\partial \delta M}{\partial q} = \frac{\partial}{\partial q} (\delta Z_w \cdot l_T) = -\frac{1}{2} \rho V S_T \left( \frac{\partial C_L^T}{\partial \alpha} + C_D^T \right) l_T^2 \quad (3.34)$$

If  $M_q$  is then made to be dimensionless, by dividing by  $1/2 \rho V S_{\text{ref}} l_{\text{ref}}^2$ , and if the drag on the tailplane is ignored (a common assumption), then Equation (3.34) reduces to Equation (3.31).

### 3.2 Lateral Coefficients

As the change in flow across any vertical fins due to a change in yaw angle  $\beta$  is analogous to a change in flow over any horizontal control surfaces due to a change in flow incidence  $\alpha$ , it is possible to rewrite all the expressions in the previous section in terms of yawed flow and vertical fins. For example,  $Y'_v$  for a vertical fin is calculated from the following expression

$$Y'_v = -[C_{Y_\beta} + C_{D_0}] \quad (3.35)$$

where  $C_{Y_\beta}$  is the lateral equivalent of  $C_{L_\alpha}$ . The derivation of the expression for  $Y'_v$  given by Equation (3.35) follows exactly the same procedure as that used in the previous section to derive  $Z'_w$ , only the forces are now resolved in the lateral plane (the x-y plane) rather than the longitudinal plane. Both Abkowitz [2] and Lewis [20] provide excellent descriptions of the methods used to calculate the four lateral coefficients  $Y_v$ ,  $Y_r$ ,  $N_v$ , and  $N_r$  for an isolated lifting surface attached to a main body. For a single fin the contribution to each of the derivatives is given as follows:

$$(Y'_r)_f = x'_f (Y'_v)_f \quad (3.36)$$

$$(N'_v)_f = x'_f (Y'_v)_f \quad (3.37)$$

$$(N'_r)_f = (x'_f)^2 (Y'_v)_f \quad (3.38)$$

where  $x'_f$  is the dimensionless axial position of the fin with respect to the centre of gravity, and  $(Y'_v)_f$  is given by Equation (3.35). It should be noted that Equations (3.36), (3.37) and (3.38) are the lateral equivalents of Equations (3.21), (3.24), and (3.30), which give the contributions of a single fin in the horizontal plane to the longitudinal coefficients  $Z_q$ ,  $M_w$  and  $M_q$ .

The above discussion has been in terms of the contribution of a single fin to the relevant hydrodynamic coefficients,  $Z_w, Z_q, M_w$  and  $M_q$  if the fin is located in the horizontal plane, or  $Y_v, Y_r, N_v$ , and  $N_r$  if the fin is located in the vertical plane. The contribution of the body itself to each of these coefficients will be discussed in the next few sections. Here however it is noted that, for an axisymmetric body, it is clear from a consideration of symmetry that the coefficients  $Y_v, Y_r, N_v$ , and  $N_r$  for the body alone are identical to the corresponding coefficients in the longitudinal plane, ie.  $Z_w, Z_q, M_w$  and  $M_q$ , apart from some changes in sign. The exact identification is as follows:

$$(Y'_v)_h = (Z'_w)_h \quad (3.39)$$

$$(N'_v)_h = -(M'_w)_h \quad (3.40)$$

$$(Y'_r)_h = -(Z'_q)_h \quad (3.41)$$

$$(N'_r)_h = (M'_q)_h \quad (3.42)$$

In Equations (3.39) to (3.42) the  $h$  subscript stands for "hull", and the equations are valid only for axisymmetric vehicles, which are the only vehicle shapes considered in this report.

## 4. The DATCOM Method

Peterson [14] wrote a technical report describing seven commonly used methods for calculating the four most common longitudinal hydrodynamic coefficients. The methods were then compared by applying them to three torpedoes and three submersibles for which experimental data were available. The seven methods considered were:

- (i) the U.S. Air Force DATCOM method.
- (ii) a semi-empirical method due to Elizabeth Dempsey of the David Taylor Naval Ship Research and Development Centre.
- (iii) a semi-empirical method derived by Strumpf at Stevens Institute of Technology, which was based on curve fits to torpedo data.
- (iv) a semi-empirical method by the Bureau of Ordnance which was derived by fitting to torpedo data.
- (v) a method due to Lanweber and Johnson at the David Taylor Model Basin, which is based on an improvement of earlier prediction methods for elongated bodies of revolution.
- (vi) a method due to Abkowitz and Paster, which is very similar to the method of Landweber and Johnson.
- (vii) a method devised by Nielsen Engineering and Research, Inc. which is based on an extensive series of wind tunnel tests for a variety of torpedo shaped underwater vehicles.

Most of the methods listed above were derived from curve fits to torpedo data. As such, they are not expected to be particularly accurate when applied to underwater vehicles of quite different shapes, such as the flatfish type vehicles. The DATCOM was based on aircraft and missile data and is, therefore, not necessarily applicable to all classes of underwater vehicles.

Peterson's application of the DATCOM method to the calculation of the hydrodynamic coefficients  $Z'_w, M'_w, Z'_q$  and  $M'_q$  for the bare hull, and  $Z'_w$  and  $M'_w$  for the bare hull plus horizontal tail configuration is summarised below. The prime notation above indicates a dimensionless coefficient, and that Peterson adopted the convention that all derivatives are non-dimensionalised with respect to the body cross-sectional area  $S_b$  and the body length  $l$ .

**4.1  $Z_w$ :** Peterson's expression for  $Z_w$  for the body alone is:

$$Z'_{w,B} = -\left(\frac{S_b}{l^2}\right) [C_{L_{\alpha,B}} + C_{D_o}] \quad (4.1)$$

where  $C_{L_{\alpha,B}}$  is the body alone lift-curve slope and  $C_{D_o}$  is the drag coefficient at zero lift. The above expression, apart from the normalisation, is the same as Equation (3.18) derived in the previous section. To apply Equation (4.1) expressions for  $C_{L_{\alpha,B}}$  and  $C_{D_o}$  are needed.

For  $C_{L_{\alpha,B}}$  Peterson used the following expression:

$$C_{L_{\alpha,B}} = 2(k_2 - k_1)S_v / S_b \quad (4.2)$$

where  $S_v$  is the effective base area, which corresponds to the area of the body at the point along the hull at which the flow becomes predominantly viscous. If distance is measured from the nose of the body then the distance  $l_v$  is computed from the expression

$$l_v = 0.378 l + 0.527 l_{ms} \quad (4.3)$$

where  $l_{ms}$  is the distance from the nose to the point of maximum slope along the afterbody and  $k_1$  and  $k_2$  are Lamb's inertial coefficients [26]. Peterson did not provide an expression for the calculation of the drag coefficient at zero lift. Equation (4.2) only applies for small angles of attack. Finck [27] provides additional techniques to calculate  $C_{L_{\alpha,B}}$  in the non-linear angle of attack range, but notes that the methods are approximate, and that each gives accurate answers only over a limited range of test conditions. Equation (4.2) is reasonable accurate for angles of attack up to approximately  $12^\circ$ , while the additional methods described by Finck [27] for the non-linear range extend the validity of the equations up to approximately  $20^\circ$ .

The tail-alone lift-curve slope is calculated using the following expression:

$$C_{L_{\alpha,T}} = \frac{2\pi AR}{2 + \sqrt{4 + (AR)^2 (1 + \tan^2 \lambda_{c/2})}} \frac{S_t}{S_b} \quad (4.4)$$

Where AR is the aspect ratio of the tail,  $\lambda_{c/2}$  is the sweep angle at the half-chord line, and  $S_t$  is the total tail planform area. The expression for the combined body/tail lift-curve slope is then

$$C_{L_{\alpha,BT}} = C_{L_{\alpha,B}} + [K_{T(B)} + K_{B(T)}]C_{L_{\alpha,T}} \quad (4.5)$$

In Equation (4.5)  $K_{T(B)}$  and  $K_{B(T)}$  are correction factors which account for the tail/body and body/tail interference effects.  $K_{T(B)}$  is the ratio of tail lift in the presence of the body to the tail lift alone, and  $K_{B(T)}$  is the ratio of the body lift in the presence of the tail to the tail lift alone. These interference factors are based on potential flow slender body theory and are functions of the tail semi-span and the radius of the body at the aerodynamic centre. The theory assumes that the tail is mounted on a slender parallel body and that viscous effects are not appreciable. Because of these assumptions the method is reasonable for forward fins on submarines, but may not be accurate for rear mounted fins. Expressions for the correction factors can be found in several references [15], [17].

The complete expression for  $Z'_w$  then becomes

$$Z'_{w,BT} = -\left(\frac{S_b}{l^2}\right) [C_{L_{\alpha,BT}} + C_{D_{\alpha,BT}}] \quad (4.6)$$

Peterson does not provide expressions for calculating  $C_{D_{\alpha,B}}$ ,  $C_{D_{\alpha,T}}$ , or  $C_{D_{\alpha,BT}}$ .

**4.2 M<sub>w</sub>:** Peterson states that the pitching moment/ angle of attack curve slope for the bare body is calculated in DATCOM by applying a viscous correction to the Munk moment,

$$C_{m_{\alpha,B}} = \frac{2(k_2 - k_1) l_v}{S_b l} \int_0^{\frac{l_v}{2}} \frac{dS}{dx} (x_m - x) dx \quad (4.7)$$

where  $x_m$  is defined as being the distance from the nose to the moment reference centre and  $l_v$  is the axial location of separation. The final expression for the pitching moment is then

$$M'_{w,B} = \left(\frac{S_b}{l^2}\right) C_{m_{\alpha,B}} \quad (4.8)$$

Equation (4.7) is a simple extension of an expression derived by Munk [25] for the lateral force per unit length for surfaces of revolution of moderate elongation moving at a small angle of attack relative to the long axis. This has the form:

$$\frac{dF}{dx} = \frac{1}{2} \rho V^2 \frac{dS}{dx} (k_2 - k_1) \sin 2\alpha \quad (4.9)$$

These lateral forces have a resulting couple, but their resultant force is zero. To calculate the moment each segment has to be multiplied by a moment arm and then integrated along the length of the body. The factor  $(x_m - x)$  indicates that the moment is taken about the centre of gravity of the body. Using the small angle approximation and then differentiating with respect to  $\alpha$ , results in the following expression for  $M_\alpha$

$$M_\alpha = \rho V^2 (k_2 - k_1) \int_0^l \frac{dS}{dx} (x_m - x) dx \quad (4.10)$$

To make Equation (4.7) non-dimensional ( $C_{m_\alpha}$ ) it is necessary to divide by  $\frac{1}{2} \rho V^2$  times a reference area times a reference length. If  $S_b$  is chosen as the reference area and  $l$  as the reference length then Equation (4.10) reduces to Equation (4.7). Other choices for the non-dimensionalisation can be made. DATCOM, for example, uses the body volume  $V_b$  instead of the product  $S_b \times l$  in order to non-dimensionalise  $M_\alpha$ .

The expression for the pitching moment coefficient *including* the contribution from the tail has the form

$$C_{m_{\alpha,BT}} = C_{m_{\alpha,B}} - C_{L_{\alpha,T}} \left[ K_{T(B)} + K_{B(T)} \right] \frac{x_t}{l} \quad (4.11)$$

where  $x_t$  is the x-coordinate of the aerodynamic centre of the tail, and the correction factors  $K_{T(B)}$  and  $K_{B(T)}$  have already been described in section 4.1. The final expression for the pitching moment coefficient  $M'_w$  then becomes

$$M'_{w,BT} = \left( \frac{S_b}{l^2} \right) C_{m_{\alpha,BT}} \quad (4.12)$$

**4.3  $Z_q$ :** Peterson considers only the contribution of the bare hull to  $Z_q$  and  $M_q$ . He quotes DATCOM as giving the bare body coefficient for lift force due to change in pitch rate as

$$C_{L_{q,B}} = C_{L_{\alpha,B}} \left( 1 - \frac{x_m}{l} \right) \quad (4.13)$$

where  $x_m$  is the distance from the nose to the moment reference centre.  $Z_q$  is then given by

$$Z'_{q,B} = C_{L_{q,B}} \times \left( -S_b / l^2 \right) \quad (4.14)$$

Comparison with Equation (3.19) shows that Equation (4.13) has a similar form, a lift coefficient times a moment arm. In Equation (3.19) however the moment arm is the distance from the centre of gravity to the tail, and the lift coefficient is that for the tail. In Equation (4.13) the moment arm is again the distance from the centre of gravity to the tail, but the lift coefficient is that for the body, indicating that the aerodynamic centre of the body is at the tail. This is unlikely for underwater vehicles, and indicates that some caution should be taken if using Equation (4.13) to calculate  $Z_q$ .

**4.4 M<sub>q</sub>:** Peterson gives the following expression for the pitching moment/pitch rate curve slope:

$$C_{m_{q,B}} = C_{m_{\alpha,B}} \frac{\left(1 - \frac{x_m}{l}\right)^2 - \frac{V}{S_{tb}l} \left(\frac{l_c}{l} - \frac{x_m}{l}\right)}{\left(1 - \frac{x_m}{l}\right) - \frac{V}{S_{tb}l}} \quad (4.15)$$

where  $l_c$  is the distance from the nose to the centre of buoyancy and  $S_{tb}$  is the cross sectional area of the truncated base. The expression for the pitching moment/pitch rate coefficient then becomes:

$$M'_{q,B} = -\frac{S_b}{l^2} C_{m_{q,B}} \quad (4.16)$$

Equation (4.15) is somewhat confusing. From the derivations presented in section 3 the following relationship between  $C_{m_{q,B}}$  and  $C_{m_{\alpha,B}}$  would be expected

$$C_{m_{q,B}} = C_{m_{\alpha,B}} \left(1 - \frac{x_m}{l}\right) \quad (4.17)$$

Equation (4.15) does not revert to this form even when the centre of buoyancy is located coincident with the centre of gravity. It is interesting to note that from Equation (4.15) if the base is not truncated, so  $S_{tb} = 0$ , and the centre of buoyancy is located coincident with the centre of gravity  $C_{m_{q,B}} = 0$ . Given that neither Peterson nor DATCOM presents the derivation of the expression given by Equation (4.15), we note that this expression should not be considered reliable until independent confirmation of its validity is obtained. Equation (4.17) also implies that the aerodynamic centre of the body is located at the tail which, as noted above, will not normally be the case for underwater vehicles.

## 5. The Roskam Method

Roskam [12] presents a very detailed prescription for the calculation of the stability derivatives for aircraft. Brayshaw [11] has presented a detailed analysis of this method and modified the techniques, where appropriate, to provide detailed expressions for the calculation of the hydrodynamic coefficients of underwater vehicles. In this section the main expressions derived by Brayshaw are outlined, and compared with the other methods described in this report.

**5.1 Z<sub>w</sub>:** The expression derived by Brayshaw [11] for  $C_{L_{\alpha, wing}}$  has the following form:

$$C_{L_{\alpha, wing}} = \frac{2\pi AR}{2 + \sqrt{4 + 4\pi^2 AR^2 (1 + \tan^2 \Lambda_{c/2}) / C_{L\alpha}^2}} \quad (5.1)$$

This shows that the expression used by Peterson for  $C_{L_{\alpha, T}}$  is actually an approximate expression which has been derived using thin wing theory,  $C_{L\alpha} = 2\pi\alpha$ . The interference of the fuselage with the wing, as well as the contribution of the fuselage itself to the lift, is taken into account using the following expression:

$$C_{L_{\alpha, wf}} = K_{wf} C_{L_{\alpha, wing}} \quad (5.2)$$

where  $K_{wf}$  is a correction factor which has the form

$$K_{wf} = 1 + 0.025 \frac{d_f}{b} - 0.25 \left( \frac{d_f}{b} \right)^2 \quad (5.3)$$

where  $d_f$  is the maximum fuselage diameter and  $b$  is the wingspan. The absence of a specific term to calculate the lift of the fuselage by itself, as per Equation (4.2) in the DATCOM method, may seem strange. The rationale behind Equation (5.2) however is that the contribution to the overall lift of an aeroplane from the fuselage is negligible compared to that from the wings, and so the small contribution from the fuselage is accounted for using the correction factor given by Equation (5.3). This reasoning is not applicable to underwater vehicles because of the relatively small size of the control surfaces, and the relatively large size of the main body. Hence the use of Equations (5.2) and (5.3) to calculate the lift of the main wing/body combination may be altered at a later stage. Any remaining horizontal control surfaces are taken into account in the final expression for  $C_{L\alpha}$  as follows:

$$C_{L\alpha} = C_{L\alpha, wing} + \sum C_{L\alpha i} \eta_i \frac{S_i}{S_{ref}} \left(1 - \frac{d\epsilon}{d\alpha}\right) \quad (5.4)$$

where  $C_{L\alpha i}$  is the lift curve slope of the control surface  $i$ ,  $S_i$  is the reference area of surface  $i$ ,  $\eta_i$  is the ratio of dynamic pressure on surface  $i$  to the free stream dynamic pressure and  $d\epsilon/d\alpha$  is the down wash gradient, which can be calculated from the following expression:

$$\frac{d\epsilon}{d\alpha} = 4.44 \left( K_A K_\lambda K_h \sqrt{\cos \Lambda_{c/4}} \right)^{1.19} \quad (5.5)$$

where  $K_A = (1/AR) - 1/(1+AR^{1.7})$ ,  $K_\lambda = (10-3\lambda)/7$ ,  $\lambda$  is the interfering wing taper ratio, and  $K_h = (1-h_h/b)/(2l_h/b)^{1/3}$ , where  $h_h$  is the height of the secondary wing with respect to the chord plane of the interfering wing, and  $l_h$  is the horizontal distance between  $1/4$  chord lines.

Methods for the calculation of the drag coefficient of an underwater vehicle are clearly explained by Brayshaw [11] and do not need to be repeated here. It should be noted however that Brayshaw, and Roskam [12], make a distinction between  $C_{D1}$ , which is the steady state drag coefficient, and  $C_{D0}$ , which is the drag coefficient when the lift is zero. Brayshaw's expression for  $Z'_w$  uses  $C_{D1}$ , while Peterson's expression uses  $C_{D0}$ , which Peterson defines as the drag coefficient at zero angle of attack. For an underwater vehicle however which has zero lift at zero angle of attack, these expressions will be the same. Hence  $Z'_w$  can be calculated from the expression

$$Z'_w = - \left( \frac{S_b}{l^2} \right) [C_{L\alpha} + C_{D1}] \quad (5.6)$$

where  $C_{L\alpha}$  is obtained from Equation (5.4), and  $C_{D1}$  is obtained from the methods described in Brayshaw [11]. Note that the expression used by Roskam to calculate  $C_{L\alpha}$  for the complete body, Equations (5.4) and (5.1), includes the lift of the fuselage in a rather non-transparent manner. A more appealing approach is to use the method derived by Munk [28], and used by both Nahon [17], and Peterson [14]. In this case, a different method would need to be used to calculate the body/wing interference factors, and this would probably be accomplished using the interference factors  $K_{T(B)} + K_{B(T)}$ .

**5.2  $M_w$ :** The expression derived by Brayshaw [11] for  $C_{M\alpha}$  from the work of Roskam [12] has the following simple form:

$$C_{M\alpha} = (\bar{x}_{re} - \bar{x}_{acA}) C_{L\alpha} \quad (5.7)$$

$C_{L_\alpha}$  is the lift/slope curve for the entire vehicle,  $\bar{x}_{ref}$  is the (dimensionless) location of the reference centre (in our case the centre of mass), and  $\bar{x}_{acA}$  is the (dimensionless) location of the aerodynamic centre of the complete vehicle. The determination of this position is a rather complicated procedure, and detailed methods are described by Brayshaw [11]. Once  $C_{M_\alpha}$  is determined then  $M'_w$  can be calculated from the expression  $M'_w = (S_b / l^2) C_{M_\alpha}$

**5.3 Z<sub>q</sub> :** Brayshaw [11] quotes the following expression for  $C_{L_q}$

$$C_{L_q} = 2 \left( C_{LH} + \sum_{i=1}^N C_{L_{q,i}} \right) \quad (5.8)$$

where

$$C_{L_{q,H}} = C_{L_{\alpha,H}} \frac{l_{hull}}{l_{ref}} \frac{S_{hull}}{S_{ref}} \quad (5.9)$$

and

$$C_{L_{q,i}} = C_{L_{\alpha,i}} \eta_i \frac{l_i}{l_{ref}} \frac{S_i}{S_{ref}} \quad (5.10)$$

where  $\eta_i$  is the slipstream interference factor. The factor of 2 in Equation (5.8) is due to the manner in which angular velocities are non-dimensionalised in aeronautical literature as previously mentioned in section 3.

**5.4 M<sub>q</sub> :** The expression given by Brayshaw for  $C_{M_q}$  is:

$$C_{M_q} = \sum_i C_{L_{q,i}} (\bar{x}_{ref} - \bar{x}_{ac_i}) + C_{L_{q,hull}} (\bar{x}_{ref} - \bar{x}_{hull}) \quad (5.12)$$

where  $\bar{x}_{ref}$  is the location of the vessel's centre of gravity,  $\bar{x}_{ac_i}$  is the location of the aerodynamic centre of control surface  $i$ , and  $\bar{x}_{hull}$  is the location of the aerodynamic centre of the hull. Equation (5.12) is a sum of terms, each of which is the product of a lift coefficient times a moment arm, and is basically an extension of the expression derived in Equation (3.35). The determination of  $\bar{x}_{hull}$  is a non-trivial exercise for a flatfish type UUV and neither Roskam nor Brayshaw provided any prescriptions to determine its value.

## 6. The University College London Method

The method described in chapter 7 of reference [15] is applicable to the calculation of the derivatives of single screw submarines. It is not claimed to be very accurate and is only applicable for small angles of incidence, and for manoeuvres in which the curvature of the path of the submarine is small. The method is intended to be used primarily for preliminary design calculations, and for estimating the effect of small changes to a design.

**6.1  $Z_w$ :** In the calculation of  $Z'_w$  it is assumed that the contribution from the hull can be neglected. It is noted that this is only true in potential flow, and that a body of revolution in a real flow at a finite angle of attack does generate lift. However, for submarines with stabilising tail surfaces the tail is normally a very efficient lifting surface compared to the hull, and so the approximation is made that

$$Z'_w = Z'_{w.TAIL} + Z'_{w.BOWFINS} \quad (6.1)$$

For a pair of fins mounted on a symmetrical body reference [15] gives the following expressions;

for an isolated wing: 
$$Z'_w = -\left(\frac{S}{l^2}\right) C_{L\alpha} \quad (6.2)$$

for a fin/body combination: 
$$Z'_w = -[K_{B(W)} + K_{W(B)}] \times \left(\frac{S}{l^2}\right) C_{L\alpha} \quad (6.3)$$

where  $K_{B(W)}$  is the ratio of lift on the body in the presence of the wing to lift on the isolated body, where the wing and the body have the same incidence, and  $K_{W(B)}$  is the ratio of the lift on the wing in the presence of the body to the lift on the isolated wing. Reference [15] gives plots of  $K_{B(W)}$  and  $K_{W(B)}$  as a function of  $r/s$ , where  $r$  is the radius of the body, and  $s$  is the distance from the centre of the body to the tip of the wing. As stated, the above definition implies that  $K_{B(W)}$  contains the contribution to  $Z'_w$  from the lift of the body. This is not the case however, and  $K_{B(W)} + K_{W(B)}$  have to be interpreted as *correction factors*. For example, in the DATCOM method as explained by Peterson, the combined body/tail lift-curve slope is given by Equation (4.5), ie.

$$C_{L_{\alpha,BT}} = C_{L_{\alpha,B}} + [K_{T(B)} + K_{B(T)}] C_{L_{\alpha,T}}$$

Here  $C_{L_{\alpha,B}}$  is the contribution to the lift of the vehicle from the body alone,  $C_{L_{\alpha,T}}$  is the contribution to the lift from the tail alone, the factor  $K_{T(B)}$  corrects for the interference to flow around the tail from the presence of the body, and the factor  $K_{B(T)}$  adds a correction to the lift of the body due to interference to the flow around the body from the effect of the tail.

Reference [15] appears to be working in the wind frame of reference and hence there is no contribution to  $Z'_w$  from drag. Reference [15], like Peterson [14], does not present any methods for the calculation of drag on the vehicle.

**6.2  $M_w$  :**  $M'_w$  is the contribution to the pitching moment due to a change in the  $z$  component of velocity. This effectively changes the lift, and hence the pitching moment about the centre of gravity. As noted in reference [15], the forces acting on a neutrally buoyant ellipsoid moving through an ideal, inviscid fluid have been calculated in several classical hydrodynamics textbooks. If the ellipsoid has an angle of attack to the flow there is no lift force, but a destabilising moment acts on the body. Lamb [26] derived the value of the moment in terms of  $k_x$  and  $k_z$ , the added mass coefficients of the body in the  $x$  and  $z$  directions, and  $m'$ , the non-dimensional mass. It has the form

$$M'_w = (k_x - k_z)m' \quad (6.4)$$

To calculate the contributions from the bowplanes and sternplanes reference [15] uses the simple prescription

$$M'_w = -Z'_w \left( \frac{x}{l} \right) \quad (6.5)$$

where  $x$  is the distance between the centre of gravity and the  $1/4$  chord position of the fin, and  $l$  is the length of the vehicle. Equation (6.5) is the product of the force (lift) times the moment arm.

**6.3  $Z_q$  :** Since it has already been assumed that the contribution to  $Z'_w$  from the hull is zero, the contribution to  $Z'_q$  from the hull will also be zero. Hence the only contributions will come from the bowplanes and sternplanes. The contribution from each of these is simply

$$Z'_q = -Z'_w \times \left( \frac{x}{l} \right) \quad (6.6)$$

which is equivalent to the expression derived in section 3, Equation (3.22). The rationale behind Equation (6.6) is that if the centre of gravity is travelling in a curved path then the incidence in the vertical plane is  $-(x/l) \times q$ . Equation (6.6) is then used to calculate  $Z'_q$  for both bowplanes and sternplanes and the two contributions are then added to give the net  $Z'_q$ . In the method described in this section, as well as in the method due to Roskam, the contribution to  $Z'_q$  from the hull is set to zero, while in Peterson's approach the only contribution to  $Z'_q$  comes from the hull.

**6.4  $M_q$  :** The calculation of  $M'_q$  is similar to the calculation of  $Z'_q$ . As the contribution to  $Z'_w$  from the hull is zero the contribution to  $M'_q$  from the hull will also be zero. Hence

the only contributions come from the bowplanes and sternplanes, and the contribution from each of these is given by

$$M'_q = Z'_w \times \left(\frac{x}{l}\right)^2 \quad (6.7)$$

Equation (6.7) is identical to Equation (3.30). Since  $Z'_w$  is negative, Equation (6.7) implies that each of the contributions to  $M'_q$  will also be negative.

## 7. Comments on Accuracy, and Sample Calculations

Peterson [14] has applied the seven different methods mentioned in section 3 to three torpedoes and three submersibles for the bare body derivatives  $Z'_w, M'_w, Z'_q$  and  $M'_q$ , and for the body/tail derivatives for  $Z'_w$  and  $M'_w$ . Each of the prediction methods was computerised and the results then compared with experimental data for the six bare body and body/tail configurations. Of particular interest to this report is the degree of accuracy shown by the various methods. Each of them poorly predicted the bare body rotary coefficient data,  $Z'_q$  and  $M'_q$ . The average error over all seven methods for  $Z'_q$  and  $M'_q$  was 60%, and the best result (Landweber and Johnson) still had an error of 27%.  $M'_w$  and  $Z'_w$  were better predicted. The average error for  $M'_w$  was 7.4%, and the average error for  $Z'_w$  was 25%. Peterson attributes the good agreement for  $M'_w$  to the fact that the modified Munk moment is a good approximation to the actual  $C_{m_\alpha}$  of slender bodies of revolution. Peterson concluded that the methods developed by NCSC in conjunction with NEAR, Inc. for computing bare hull static normal force and pitching moment coefficients gave the best overall results, predicting within 22% of the data on all six configurations. The NEAR method is the one programmed into Peterson's HYSUB code for the hydrodynamic analysis of submersibles [13].

Reference [15] provides a worked example for the calculation of  $Z'_w, M'_w, Z'_q$  and  $M'_q$  for a 61.0 m submarine. This example also includes contributions to the hydrodynamic coefficients from the effects of vortices over the hull and sternplanes, as well contributions from the propeller. The magnitude of the various contributions are shown in Table 1. The relative magnitudes and signs of the separate contributions to each of the coefficients is quite interesting. For  $M'_w$ , for example, the largest contribution comes from the hull and has a positive sign, indicating that it is a destabilising moment. The second largest contribution comes from the sternplanes. Longitudinal stability of an aeroplane requires that  $M'_w$  be negative. On an aeroplane the main wing is often slightly forward of the centre of gravity and hence makes a positive contribution to  $M'_w$  (as do the bowplanes in

the above example). Longitudinal stability is provided by the tail, where the product of the lift on the tail, times the long moment arm, provides a negative contribution which overcomes the positive contribution from the main wing. For underwater vehicles, such as a submarine, the situation is quite different. The main contribution to  $M'_{yw}$  comes from the hull. The sternplanes provide a negative contribution, but  $M'_{yw}$  still has a relatively large positive value.

Reference [15] also considers the contribution from the propeller, and the effect of the interaction of the vortices with the sternplanes and hull, to the final values for the hydrodynamic coefficients. It can be seen that the hull, bowplanes and sternplanes provide the largest contributions to  $M'_{yw}$ , accounting for 73% of the total value of  $M'_{yw}$ .

Table 1. Hydrodynamic Coefficients for 61 metre submarine [15].

| Component               | $Z'_w (\times 10^3)$ | $M'_{yw} (\times 10^3)$ | $Z'_q (\times 10^3)$ | $M'_q (\times 10^3)$ |
|-------------------------|----------------------|-------------------------|----------------------|----------------------|
| Hull                    | 0                    | +13.88                  | 0                    | 0                    |
| Propeller               | -1.32                | -0.70                   | -0.70                | -0.37                |
| Bowplane                | -11.34               | +3.74                   | +3.74                | -1.23                |
| Sternplane              | -23.04               | -10.60                  | -10.60               | -4.88                |
| Total exc. vortices     | -35.70               | +6.32                   | -7.56                | -6.48                |
| Vortices on Sternplanes | +6.60                | +3.04                   | -2.18                | -1.00                |
| Vortex on Hull          | -2.62                | -0.84                   | +0.86                | +0.28                |
| Total inc. vortices     | -31.72               | +8.52                   | -8.88                | -7.20                |

As a further test of the accuracy of some of these algorithms the DATCOM method was used to calculate  $Z_w$ ,  $M_w$ ,  $Z_q$ , and  $M_q$  for four different torpedo shapes. The algorithms were coded using the MATLAB package and the torpedo data was taken from the Hydroballistics Design Handbook [29]. The torpedo shapes were specified in the Handbook by listing the diameter values at the corresponding axial positions. All of the experimental results quoted here are for torpedos with bare hulls. In this case the body is axisymmetric and the longitudinal coefficients  $Z_w$ ,  $M_w$ ,  $Z_q$ , and  $M_q$  are equal to the lateral coefficients  $Y_v$ ,  $N_v$ ,  $Y_r$ , and  $N_r$ , hence only experimental values for the longitudinal coefficients need be calculated.

One of the problems with making a comparison between calculated and experimental values for the hydrodynamic coefficients found in the Hydroballistic Design Handbook is that the method used to non-dimensionalise the coefficients has not been explicitly stated. It appears, for example, that the experimental value of  $Z_w$  has been non-dimensionalised

by dividing the dimensional result by the factor  $\rho VS_{\text{ref}}$ , rather than the factor  $\frac{1}{2}\rho VS_{\text{ref}}$ . Using this convention, the results shown in Tables 2 through 5 were obtained. The calculated values are shown in the columns labelled HYGUESS, which is the name of the computer program which calculated these results.

For the Mark 13 and Mark 18 torpedos the calculated values are in quite good agreement with the measured values, with both  $Z'_w$  and  $M'_w$  agreeing to within a few percent with the experimental results. Only  $Z'_q$  for the Mark 18 torpedo shows a significantly larger error of 38%. For the Mark 36 and Mark 41 torpedos the agreement is not quite as good, but still within acceptable limits for these types of algorithms, as noted by Peterson [14]. The results for  $Z'_w$  and  $M'_w$  agree with the experimental values to within 26%, while the results for  $Z'_q$  and  $M'_q$  have mismatches of between 16% and 37%.

Table 2: Comparison of calculated and experimental values for the four longitudinal hydrodynamic coefficients  $Z'_w$ ,  $M'_w$ ,  $Z'_q$ , and  $M'_q$  for the Mark 13 Torpedo.

| MARK 13 MOD TORPEDO |         |            |                       |
|---------------------|---------|------------|-----------------------|
| Coefficient         | HYGUESS | Experiment | Percentage difference |
| $Z'_w$              | -0.593  | -0.60      | 1.2                   |
| $M'_w$              | 0.9932  | 0.99       | 0.0                   |
| $Z'_q$              | -0.209  | -0.20      | 5.0                   |
| $M'_q$              | -0.0740 | -0.08      | 7.5                   |

Table 3: Comparison of calculated and experimental values for the four longitudinal hydrodynamic coefficients  $Z'_w$ ,  $M'_w$ ,  $Z'_q$ , and  $M'_q$  for the Mark 18 Torpedo.

| MARK 18 MOD TORPEDO |         |            |                       |
|---------------------|---------|------------|-----------------------|
| Coefficient         | HYGUESS | Experiment | Percentage difference |
| $Z'_w$              | -0.779  | -0.76      | 2.6                   |
| $M'_w$              | 1.056   | 1.094      | 3.4                   |
| $Z'_q$              | -0.284  | -0.206     | 37.9                  |
| $M'_q$              | -0.1037 | -0.117     | 11.4                  |

Table 4: Comparison of calculated and experimental values for the four longitudinal hydrodynamic coefficients  $Z'_w$ ,  $M'_w$ ,  $Z'_q$ , and  $M'_q$  for the Mark 36 Torpedo.

| MARK 36 MOD TORPEDO |         |            |                       |
|---------------------|---------|------------|-----------------------|
| Coefficient         | HYGUESS | Experiment | Percentage difference |
| $Z'_w$              | -0.918  | -0.94      | 2.1                   |
| $M'_w$              | 0.960   | 1.156      | 17.0                  |
| $Z'_q$              | -0.324  | -0.384     | 15.6                  |
| $M'_q$              | -0.114  | -0.181     | 37.0                  |

Table 5: Comparison of calculated and experimental values for the four longitudinal hydrodynamic coefficients  $Z'_w$ ,  $M'_w$ ,  $Z'_q$ , and  $M'_q$  for the Mark 41 Torpedo.

| MARK 41 MOD TORPEDO |         |            |                       |
|---------------------|---------|------------|-----------------------|
| Coefficient         | HYGUESS | Experiment | Percentage difference |
| $Z'_w$              | -0.584  | -0.68      | 14.7                  |
| $M'_w$              | 0.73022 | 0.991      | 26.0                  |
| $Z'_q$              | -0.210  | -0.16      | 31.0                  |
| $M'_q$              | -0.0757 | -0.11      | 31.0                  |

## 8. Discussion and Conclusion

A great deal of the literature which describes methods for the calculation of hydrodynamic derivatives is based on methods which are applicable to standard aeroplane designs. Adapting these prescriptions for use with underwater vehicles has led to problems in calculating some of the derivatives because of significant differences between the basic shapes of aeroplanes and underwater vehicles. In particular, an aeroplane typically has a single large wing area which generates virtually all the lift on the vehicle. Underwater vehicles, on the other hand, typically have very small control surfaces fore and aft which generate minimal lift. For an AUV the lift generated by the hull may well be comparable to that generated from the fins. This is in contrast to an aeroplane, where the lift from the fuselage may either be neglected entirely, or treated in a very superficial manner.

Another significant difference between aeroplanes and underwater vehicles occurs in the basic shape of the main body. For aeroplanes the fuselage is usually cylindrical or closely resembles a cylinder, and the methods for the calculation of the derivatives in many cases are based on this implicit assumption. While some underwater vehicles, for example the ARCS vehicle [30], have cylindrical hulls, many, such as Marius [31] and Wayamba [32], are based on a flatfish design, and methods based on cylindrical hulls are inappropriate for these vehicles.

Given the problems referred to above, the simplified approach pioneered by Nahon [17] is attractive because it avoids the calculation of the hydrodynamic derivatives and calculates the hydrodynamic forces directly by summing components of the lift and drag forces on the main hull and all control surfaces and appendages. As previously noted however, a drawback with the method as described by Nahon is the very rudimentary manner in which interference effects are taken into account. Nevertheless, the method has an appealing simplicity, and it may be possible to pursue this approach further by incorporating some of the more detailed methods described above which provide more accurate expressions for calculating the correction factors.

An alternative approach to the calculation of hydrodynamic coefficients for non axisymmetric bodies is to combine experimental techniques with current Computational Fluid Dynamics (CFD) capabilities. In the last two decades both the sophistication of CFD codes, and the computing power of standard desk top workstations, have increased significantly. The possibility of using CFD to determine hydrodynamic derivatives is now just becoming feasible [33]. In MPD we intend to use both axisymmetric and non-axisymmetric scale models in the experimental facilities at the Australian Maritime Engineering College in Launceston to measure hydrodynamic coefficients for a variety of underwater vehicle shapes. These results will then be used to benchmark simulation results for these scale models from the Fluent CFD code. Provided the level of agreement between the simulation results and the experimental results is reasonable, we will then use Fluent to perform a parametric study on a variety of UUV shapes to determine the manoeuvrability characteristics of each of these vehicles.

## 9. References

- [1] Rocard, S. "Formulation des equations couplees des mouvements d'un corps solide remorque dans un fluide illimite et de son cable", SIREHNA Internal Report, July 1990.
- [2] Abkowitz, M. A. "Stability and Motion Control of Ocean Vessels", M.I.T. Press, Massachusetts Institute of Technology, 1969.
- [3] Anderson, B. "Analysis of Planar Motion Mechanism Data for Manoeuvring Simulations of Unmanned Underwater Vehicles", Masters Thesis, University of Tasmania, December 1998.
- [4] Brutzman, D.P., "A Virtual World for an Autonomous Underwater Vehicle", Ph.D. Thesis, Naval Postgraduate School, Monterey, December 1994.
- [5] Gertler, M and Hagen , G.R. "Standard Equations of Motion for Submarine Simulation", Naval Ship Research and Development Centre, Washington, DC, June, 1967.
- [6] Strumpf, A. "Equations of Motion of a Submerged Body with Varying Mass", Stevens Institute of Technology, Report SIT-DL-60-9-771, May 1960.
- [7] Clarke, D. Unpublished notes on symmetry properties of hydrodynamic coefficients, October, 1998.
- [8] Etkin, B. "Dynamics of Atmospheric Flight", John Wiley & Sons, Inc., 1972, p. 159.
- [9] Russell, J.B., "Performance and Stability of Aircraft", Arnold, 1996.
- [10] Blakelock, J. A. "Automatic Control of Aircraft and Missiles", Second Edition, John Wiley & Sons, Inc., 1991.
- [11] Brayshaw, I. "Hydrodynamic Coefficients of Underwater Vehicles", Vacation Student Report, Maritime Platforms Division, Aeronautical and Maritime Research Laboratories, DSTO, Melbourne, 1999.
- [12] Roskam, J. "Airplane Design - Part VI: Preliminary Calculation of Aerodynamic, Thrust and Power Characteristics", Roskam Aviation and Engineering Corporation, Kansas, 1990.
- [13] Peterson, R.S. "Hydrodynamic Analysis of Submersibles - The HYSUB System", March, 1997.
- [14] Peterson, R.S. "Evaluation of semi-empirical methods for predicting linear static and rotary hydrodynamic coefficients", NCSC TM 291-80.

- [15] Author unknown "Calculation of Submarine Derivatives". University College London Postgraduate Submarine Design Notes, 1988.
- [16] Nahon, M. "Determination of Undersea Vehicle Hydrodynamic Derivatives Using the USAF Datcom", 1993, pp. II-283 – II-288.
- [17] Nahon, M., "A Simplified Dynamics Model for Autonomous Underwater Vehicles", AUV Technology 1996, pp. 373-379.
- [18] Wolkerstorfer, W.J, "A Linear Manoeuvring Model for Simulation of SLICE Hulls", Master Thesis, Naval Postgraduate School, Monterey, CA, 1995.
- [19] Holmes, E.P., "Prediction of Hydrodynamics Coefficients Utilising Geometric Considerations", Masters Thesis, Naval Postgraduate School, Monterey, CA, Dec. 1995.
- [20] Lewis, E.V., ed. "Principles of Naval Architecture, Volume 3", published by Society of Naval Architects and Marine Engineers (SNAME), 1989.
- [21] Babister, "Aircraft Stability and Control", Pergamon Press, 1961.
- [22] Smetana, F.O. "Computer Assisted Analysis of Aircraft Performance Stability and Control", McGraw-Hill, 1984.
- [23] McCormack, B.W. "Aerodynamics, Aeronautics and Flight Mechanics", Second Edition, John Wiley and Sons, 1995.
- [24] Houghton, E.L. and Carpenter, P.W. "Aerodynamics for Engineering Students", 4<sup>th</sup> edition, Edward Arnold, 1993.
- [25] Clancy, L.J. "Aerodynamics", Pitman Books Ltd., 1975.
- [26] Lamb, H. "Hydrodynamics", Cambridge University Press, 6<sup>th</sup> edition, 1975.
- [27] Finck, R.D. "USAF Stability and Control Data Compendium", (DATCOM), Air Force Flight Dynamics Laboratory, Wright Patterson Air Force Base, April, 1976.
- [28] Munk, M.M. "Aerodynamic Theory" Vol.1 and Vol. 6, Durand, W.F. editor-in-chief, 1934.
- [29] Hydroballistics Design Handbook, Volume 2, Ed. R.J. Grady, Naval Sea Systems Command, USN, SEAHAC TR 79-1, CONFIDENTIAL, October 1997 .
- [30] Hopkin, D. and den Hertog, V., "The Hydrodynamic Testing and Simulation of an Autonomous Underwater Vehicle", Proceedings of the Second Canadian Marine Dynamics Conference, pp. 274-281, 1993.
- [31] Aage, C. and Smitt, L.W., "Hydrodynamic Manoeuvrability Data of a Flatfish Type AUV", IEEE Proceedings, OCEANS 94, pp. III-425 to III-430.
- [32] Janes Underwater Technology, 5<sup>th</sup> Edition, Ed. Clifford Funnell, page 3, 2002-2003.
- [33] Nahon, M., private communication, June, 1999.

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## 19. ABSTRACT

Maritime Platforms Division within DSTO is currently studying the emerging science and technology of autonomous underwater vehicles for defence applications. As part of an examination of the requirements for the hydrodynamics and manoeuvrability of these vehicles MPD has been tasked with the development of models to determine the hydrodynamic coefficients of simple and complex submerged bodies as a function of their shape. These coefficients are specific to the vehicle and provide the description of the hydrodynamic forces and moments acting on the vehicle in its underwater environment. This report provides a detailed discussion and evaluation of three of the existing methods which have been documented in the literature for the calculation of these coefficients. Sample calculations using some of these techniques are presented, and the accuracy and applicability of these calculational methods to the underwater vehicles of interest to the DSTO are described. It is concluded that none of the methods surveyed has the necessary generality to encompass all the shapes of interest to DSTO work, and alternative computational techniques are recommended which should allow the hydrodynamic coefficients of more complex underwater vehicles to be determined.